

# CQS-Pair: Cyclic Quorum System Pair for Wakeup Scheduling in Wireless Sensor Networks

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**Abstract.** Due to the heterogenous power-saving requirement in wireless sensor networks, we propose the Cyclic Quorum System Pair (CQS-Pair) which can guarantee that two asynchronous nodes adopt different cyclic quorum systems can hear each other at least once in bounded time intervals. To quickly assemble a CQS-Pair, we present a fast construction scheme, which is based on the Multiplier Theorem and the  $(N, k, M, l)$ -difference pair defined by us. We show that via the CQS-Pair, two heterogenous nodes can achieve different power saving ratios while maintaining connectivity. The performance of a CQS-Pair is analyzed in terms of average delay, quorum ratio, and issues for supporting multicast/broadcast.

## 1 Introduction

Wireless sensor networks have recently received increased attention for a broad array of applications such as surveillance, environment monitoring, medical diagnostics, and industrial control. As wireless sensor nodes usually rely on portable power sources such as batteries to provide the necessary power, their power management has become a crucial issue. It has been observed that idle energy plays an important role for saving energy in wireless sensor networks [2]. Most existing radios (i.e., CC2420) used in wireless sensor networks support different modes, like transmit/receive mode, idle mode, and sleep mode. In idle mode, the radio is not communicating but the radio circuitry is still turned on, resulting in energy consumption which is only slightly less than that in the transmitting or receiving states.

In order to save idle energy, it is necessary to introduce a wakeup mechanism for sensor nodes in the presence of pending transmission. The major objective of the wakeup mechanism is to maintain network connectivity while reducing the idle state energy consumption. Existing wakeup mechanisms fall into three categories: on-demand wakeup, scheduled rendezvous, and asynchronous wakeup.

In on-demand wakeup mechanisms [6], out-band signaling is used to wake sleeping nodes in an on-demand manner. For example, with the help of a paging signal, a node listening on a page channel can be woken up. As page radios can operate at lower power consumption, this strategy is very energy efficient. However, it suffers from increased implementation complexity.

In scheduled rendezvous wakeup mechanisms, low-power sleeping nodes wake up at the same time periodically to communicate with one another. One example is the S-MAC [11] protocol.

The third category, asynchronous wakeup [12], is also well studied. Compared to the scheduled rendezvous wakeup mechanisms, asynchronous wakeup does not require clock synchronization. Each node follows its own wakeup schedule in idle state, as long as the wakeup intervals among neighbors overlap. To meet this requirement, nodes usually have to wakeup more frequently than in the scheduled rendezvous mechanisms. But the advantage of asynchronous wakeup is the easiness in implementation. Furthermore, it can ensure network connectivity even in highly dynamic networks.

The quorum-based wakeup scheduling paradigm, also called quorum-based power saving (QPS) protocol [3, 10], has recently received significant attentions as an asynchronous wakeup solution. In

a QPS protocol, the time axis on each node is evenly divided into beacon intervals. Given an integer  $n$ , a quorum system defines a cycle pattern, which specifies the awake/sleep scheduling pattern during  $n$  continuous beacon intervals for each node. We call  $n$ , the *cycle length*, since the pattern repeats every  $n$  beacon intervals. A node may stay awake or sleep during each beacon interval. QPS protocols can guarantee that at least one awake interval overlaps between two adjacent nodes with only  $O(\sqrt{n})$  beacon intervals being awake in each node. Most previous work only consider homogeneous quorum systems for asynchronous wakeup scheduling, which means that quorum systems for all nodes have the same *cycle length*.

However, it is often desirable that heterogeneous nodes (i.e, clusterheads and members) have heterogeneous quorum-based wakeup schedule (or different *cycle lengths*). We denote two quorums from different quorum systems as *heterogeneous quorums* in this paper. If two adjacent nodes adopt heterogeneous quorums as their wakeup schedules, they have different cycle lengths and different wakeup patterns. The problem is how to guarantee that the two nodes can discover each other within bounded delay in the presence of clock drift.

In this paper, we present the Cyclic Quorum System Pair (CQS-Pair) which contains a pair of quorum systems suitable for heterogeneous quorum-based wakeup schedule. The mechanism of CQS-Pair can guarantee that two adjacent nodes adopt heterogeneous quorums from such a pair as their wakeup schedule, can hear each other at least once within one super cycle length (i.e., the bigger cycle length in the CQS-Pair). With the help of the CQS-Pair, wireless sensor networks can achieve better trade-off between energy consumption and average delay. For example, all clusterheads and gateway nodes can pick up a quorum from the quorum system with shorter cycle length as their wake up schedule, to get shorter discovery delay. In addition, all members in a cluster can choose a quorum from the system with longer cycle length as their wakeup schedules, in order to save more idle energy.

**Our contribution.** We present the Cyclic Quorum system Pair (CQS-Pair) and propose a fast constructing scheme via the Multiplier Theorem and  $(N, k, M, l)$ -difference pair defined by us. We show that the CQS-Pair has the heterogeneous-rotation closure property (defined in Section 3.2) and can be applied as a solution for the problem of heterogeneous quorum-based wakeup scheduling in wireless sensor networks. The CQS-Pair is also an optimal design in terms of quorum ratios, given a pair of cycle lengths ( $n$  and  $m$ ,  $n \leq m$ ). We also analyze its performance in terms of expected delay ( $\frac{n-1}{2} < E(\text{delay}) < \frac{m-1}{2}$ ), quorum ratio, energy saving ratio, and practical issues on how to support multicast/broadcast. This is the first solution to heterogeneous quorum-based wakeup scheduling as we are not aware of any other existed similar solutions.

*Paper Structure.* The rest of the paper is organized as follows: In Section 2, we outline some basic preliminaries for quorum-based power-saving protocols. The detailed design and construction scheme of Cyclic Quorum System Pair is discussed in Section 3. We analyze the performance of the CQS-Pair in Section 4. Related work is presented in Section 5. We conclude in Section 6.

## 2 Preliminaries

### 2.1 Network Model and Assumptions

We represent a multi-hop wireless sensor network by a directed graph  $G(V, E)$ , where  $V$  is the set of network nodes ( $|V| = N$ ), and  $E$  is the set of edges. If node  $v_j$  is within the transmission range of node  $v_i$ , then an edge  $(v_i, v_j)$  is in  $E$ . We assume bidirectional links. The major objective of quorum-based wakeup scheduling is to maintain network connectivity regardless of clock drift. Here we use the term ‘‘connectivity’’ loosely, in the sense that a topologically connected network in our context may not be connected at any time; instead, all the nodes are reachable from a node within a finite amount of time.

We also make the following assumptions when applying quorum-based system for asynchronous wakeup mechanism: 1) All time slots have equal length, being 1 in this paper for convenient

presentation; and 2) The overhead of turning on and shutting down radio is negligibly small. The length of one time slot depends on application-specific requirements. For a radio compliant with IEEE 802.15.4, the bandwidth is approximately 128kb/s and a typical packet size is 512KB. Given this, the slot length (beacon interval) is  $\leq 50ms$ . The second assumption, also adopted by previous work [12] [3], is for the convenience of theoretical analysis.

## 2.2 Heterogeneous Quorum-Based Power Saving in Sensor Networks

In sensor networks, it is often desirable that different nodes wakeup in heterogeneous quorum-based schedules. First, many wireless sensor networks have heterogeneous entities, like cluster-heads, gateways, member nodes, and relay nodes. They have different requirements regarding average neighbor discovery delay and energy saving ratio. Regarding cyclic quorum systems, generally, cluster-heads should wakeup based a quorum system with short cycle length, and member nodes with longer cycle length. Second, wireless sensor networks that are used in applications such as environment monitoring typically have seasonally-varying power saving requirements. For example, a sensor network for wild fire monitoring may require a larger energy saving ratio during winter seasons. Thus, they often desire variable cycle-length wakeups in different seasons.

Previous work [3] defined the QPS (quorum-based power-saving) problem as follows: Given an universal set  $U = \{0, 1, \dots, n-1\}$  ( $n > 2$ ) and a quorum system  $\mathcal{Q}$  over  $U$ , two nodes picks up any quorum from  $\mathcal{Q}$  as their wakeup schedule must have at least one overlap in every  $n$  consecutive slots. It has been shown in [3] that the cyclic quorum system and grid quorum system are both solutions to the QPS problem.

However, when there are two heterogenous quorum systems  $\mathcal{X}$ ,  $\mathcal{Y}$ , and two heterogenous quorums  $G \in \mathcal{X}$ ,  $H \in \mathcal{Y}$ , it is more difficult to guarantee that two nodes picking up  $G$  and  $H$  as their wakeup schedules respectively can hear each other within bounded time delay. Considering the two heterogeneous quorum systems  $\mathcal{X}$  over  $\{0, 1, \dots, n-1\}$  and  $\mathcal{Y}$  over  $\{0, 1, \dots, m-1\}$  ( $n \leq m$ ), we define the **h-QPS** problem as follows: design a pair  $(\mathcal{X}, \mathcal{Y})$  in order to guarantee that two nodes picking up  $G \in \mathcal{X}$  and  $H \in \mathcal{Y}$  as their wakeup schedules respectively can hear each other at least once within every  $m$  consecutive slots, and that  $\mathcal{X}$  and  $\mathcal{Y}$  are solutions to QPS individually.

A solution to the h-QPS problem is important to keep connectivity when we want to dynamically change the quorum systems between all nodes. Suppose all nodes in a network initially wakeup via a longer cycle length. When there is a need to reduce the cycle length (i.e., to meet a delay requirement or vary with changing seasons), the sink node can send a request to the whole network gradually through some relay nodes. If a relay node changes its quorum system first, the remaining nodes and the relay node will loose connectivity if two kinds of nodes that wakeup based on heterogenous quorums cannot hear each other at least once in every  $m$  consecutive slots.

## 2.3 Quorum-based Power-Saving Protocols

We use the following definitions of quorum system. Given a cycle length  $n$ , let  $U = \{0, 1, \dots, n-1\}$  be an universal set.

**Definition 1.** A quorum system  $\mathcal{Q}$  under  $U$  is a collection of non-empty subsets of  $U$ , each called a quorum, which satisfies the intersection property:  $\forall G, H \in \mathcal{Q} : G \cap H \neq \emptyset$ .

**Definition 2.** Given an integer  $i \geq 0$  and quorum  $G$  in a quorum system  $\mathcal{Q}$  under  $U$ , we define  $G + i = \{(x + i) \bmod n : x \in G\}$ .

**Definition 3.** A quorum system  $\mathcal{Q}$  under  $U$  is said to have the rotation closure property if  $\forall G, H \in \mathcal{Q}, i \in \{0, 1, \dots, n-1\} : G \cap (H + i) \neq \emptyset$ .

**Theorem 1.**  $\mathcal{Q}$  is a solution to the QPS problem if  $\mathcal{Q}$  is a quorum system satisfying the rotation closure property.

There are two kinds of quorum systems, grid quorum system and cyclic quorum system, that satisfy the rotation closure property and can be applied for asynchronous wakeup in wireless sensor networks.

**Grid-Quorum System** [5]. In a grid quorum system shown in Figure 1, elements are arranged as a  $\sqrt{n} \times \sqrt{n}$  array (square). A quorum can be any set containing a column and a row of elements in the array. The quorum size in a square grid quorum system is  $2\sqrt{n} - 1$ . An alternative is a “triangle” grid-based quorum in which all elements are organized in a “triangle” fashion. The quorum size in “triangle” quorum system is approximately  $\sqrt{2}\sqrt{n}$ .

**Cyclic Quorum System** [5]. A cyclic quorum system is based on the ideas of cyclic block design and cyclic difference sets in combinatorial theory [7]. The solution set can be strictly symmetric for arbitrary  $n$ . For example,  $\{1, 2, 4\}$  is a quorum from the cyclic quorum system with cycle length=7. Figure 1 illustrates three quorums from a cyclic quorum system with cycle length 7.

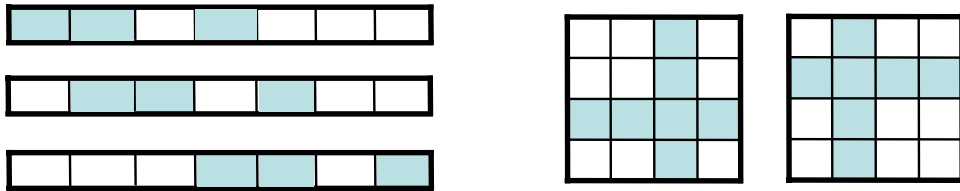


Fig. 1. Cyclic Quorum System (left) and Grid Quorum System (right)

**Theorem 2.** Both grid-quorum systems and cyclic quorum systems satisfy the rotation closure property and can be applied for QPS in wireless sensor networks.

**Theorem 3.** The bound of quorum size  $k$  for a quorum system  $Q$  over  $N$  is  $k \geq \sqrt{n}$ .

Proofs of Theorems 1, 2, and 3 can be found in [3]. Generally, two quorums  $G$  and  $H$  from a cyclic quorum system or a grid quorum system are *homogenous* with each other since they have the same quorum size and  $G = H + i$  which shows a rotation property. Although grid-quorum systems and cyclic quorum systems can be applied as a solution for QPS problem, it is not necessary meaning that they can be a solution to the h-QPS problem. We will show this in Section 3.

## 2.4 Neighbor Discovery

Since sensor nodes are subject to clock drift, we cannot assume that time slots are exactly aligned to their boundaries. But with the quorum-based wakeup schedule, we can ensure that there exists sufficient overlapping active slots even if two schedules have non-synchronized clocks with respect to each other.

**Theorem 4.** If two quorums ensure a minimum of one overlapping slot, then with the help of a beacon message at the beginning of each slot, two neighboring nodes can hear each others' beacons with at least once.

Theorem 4's proof is presented in [12]. This theorem ensures that two neighboring nodes can always discover each other within bounded time if all beacon messages are transmitted successfully. This property also holds true even in the case that two originally disconnected subsets of nodes join together as long as they use the same quorum system.

## 3 Cyclic Quorum System Pair

In this section, we present the CQS-Pair which is based on the cyclic block design and cyclic difference sets in combinatorial theory [7]. CQS-Pair can be applied as a solution to the h-QPS problem.

### 3.1 Heterogeneous Rotation Closure Property

First, we define some concepts to facilitate our presentation. Some definitions are extended from those in [5]. We will also use definitions from [7] to denote  $\mathbb{Z}_n$  as an finite field of order  $n$  and  $(\mathbb{Z}_n, +)$  as an Abelian Group.

**Definition 4.** Let  $A$  be a set in  $(\mathbb{Z}_n, +)$ . For any  $g \in \mathbb{Z}_n$ , we define  $A+g = \{(x+g) \bmod n : x \in A\}$ .

**Definition 5.** (Cyclic set) Let  $X$  be a set in  $(\mathbb{Z}_n, +)$ . The set  $C(X, \mathbb{Z}_n)$  is called a cyclic set (or cyclic group) of  $X$  if  $C(X, \mathbb{Z}_n) = \{X + i \mid i \in \mathbb{Z}_n\}$ .

**Definition 6.** ( $p$ -extension) Given two positive integers  $n$  and  $p$ , suppose  $U = \{0, 1, \dots, n-1\}$  and let  $U^p = \{0, \dots, p*n-1\}$ . For a set  $A = \{a_i \mid 1 \leq i \leq k, a_i \in U\}$ ,  $A$ 's  $p$ -extension is defined as  $A^p = \{a_i + j*n \mid 1 \leq i \leq k, 0 \leq j \leq p-1, a_i \in U\}$  over  $U^p$ . For a cyclic quorum system  $\mathcal{Q} = \{A, A+1, \dots, A+n-1\}$  over  $U$ ,  $\mathcal{Q}$ 's  $p$ -extension is defined as  $\mathcal{Q}^p = \{A^p, (A+1)^p, \dots, (A+n-1)^p\}$  over  $U^p$ .

For example, if  $A = \{1, 2, 4\}$  in  $(\mathbb{Z}_7, +)$ ,  $A^3 = \{1, 2, 4, 8, 9, 11, 15, 16, 18\}$  in  $(\mathbb{Z}_{21}, +)$ . If a quorum system  $\mathcal{Q} = \{\{1, 2, 4\}, \{2, 3, 5\}, \{3, 4, 6\}\}$ , we have  $\mathcal{Q}^2 = \{\{1, 2, 4, 8, 9, 11\}, \{2, 3, 5, 9, 10, 12\}, \{3, 4, 6, 10, 11, 13\}\}$ .

**Definition 7.** (Heterogeneous rotation closure property) Given two positive integers  $N$  and  $M$  where  $N \leq M$  and  $p = \lceil \frac{M}{N} \rceil$ , consider two quorum systems  $\mathcal{X}$  over the universal set  $\{0, \dots, N-1\}$  and  $\mathcal{Y}$  over the universal set  $\{0, \dots, M-1\}$ . Let the quorum system  $\mathcal{X}$ 's  $p$ -extension be denoted as  $\mathcal{X}^p$ . The pair  $(\mathcal{X}, \mathcal{Y})$  is said to satisfy the heterogeneous rotation closure property if :

1.  $\forall G \in \mathcal{X}^p, H \in \mathcal{Y}, i \in \{0, \dots, M-1\} : G \cap (H+i) \neq \emptyset$ , and
2.  $\mathcal{X}$  and  $\mathcal{Y}$  satisfy the rotation closure property, respectively.

Consider two sets  $A = \{1, 2, 4\}$  in  $(\mathbb{Z}_7, +)$  and  $B = \{1, 2, 4, 10\}$  in  $(\mathbb{Z}_{13}, +)$ . If two cyclic quorum systems  $\mathcal{Q}_A = C(A, \mathbb{Z}_7)$  and  $\mathcal{Q}_B = C(B, \mathbb{Z}_{13})$ , then  $\mathcal{Q}_A^2 = C(\{1, 2, 4, 8, 9, 11\}, \mathbb{Z}_{14})$ . We can verify that any two quorums from  $\mathcal{Q}_A^2$  and  $\mathcal{Q}_B$  must have an intersection. Thus, the pair  $(\mathcal{Q}_A, \mathcal{Q}_B)$  satisfies the heterogeneous rotation closure property.

**Lemma 1.** If two quorum systems  $\mathcal{X}$  and  $\mathcal{Y}$  satisfy the heterogeneous closure property, then the pair  $(\mathcal{X}, \mathcal{Y})$  is a solution to the h-QPS problem.

*Proof.* According to the Definition 7, if two quorum systems  $\mathcal{X}$  and  $\mathcal{Y}$  satisfy the heterogeneous closure property, a quorum  $G$  from  $\mathcal{X}$  and a quorum  $H$  from  $\mathcal{Y}$  must overlap at once within the bigger cycle length of  $\mathcal{X}$  and  $\mathcal{Y}$ . So two nodes can hear each other if they pick up  $G$  and  $H$  as wakeup schedule based on the Theorem 4. It indicates that  $(\mathcal{X}, \mathcal{Y})$  is a solution to the h-QPS problem.

**Lemma 2.** Let Grid Quorum System-Pair be defined as a pair consisting of two grid quorum systems. Then Grid Quorum System-Pair satisfies the heterogeneous rotation closure property and can be a solution to the h-QPS problem.

*Proof.* It has been proven in [3] that the grid quorum system satisfies the rotation closure property. Thus, we only need to prove that for two grid quorum systems  $\mathcal{X}$  over  $\{0, \dots, n-1\}$  and  $\mathcal{Y}$  over  $\{0, \dots, m-1\}$  ( $n \leq m, p = \lceil \frac{m}{n} \rceil$ ),  $\forall G^p \in \mathcal{X}^p, H \in \mathcal{Y}, i \in \{0, \dots, M-1\}$ , there is  $G \cap (H+i) \neq \emptyset$ . It is equal to prove that  $(G+i)^p \cap H \neq \emptyset$ . Consider a quorum  $G$  from  $\mathcal{X}$  which contains all elements on the column  $c$ , namely  $c, c+\sqrt{n}, \dots, c+\sqrt{n}(\sqrt{n}-1)$ , where  $0 \leq c < \sqrt{n}$ . Then, a quorum  $(G+i)^p$  from the  $p$ -extension of  $\mathcal{X}$  contains elements which has the largest distance of  $\sqrt{n}$  between any two consecutive elements. It must have an intersection with  $H$  since  $H$  contains a full row which has  $\sqrt{m}$  ( $\geq \sqrt{n}$ ) consecutive integers. Thus, Grid Quorum System-Pair satisfies the heterogeneous rotation closure property and can be a solution to the h-QPS problem.

### 3.2 $(N, k, M, l)$ -difference pair

**Definition 8.**  $(N, k, M, l)$ -*difference pair*. Suppose  $N \leq M$  and  $p = \lceil \frac{M}{N} \rceil$ . Consider two sets  $A : \{a_1, \dots, a_k\}$  in  $(\mathbb{Z}_N, +)$  and  $B : \{b_1, \dots, b_l\}$  in  $(\mathbb{Z}_M, +)$ . The pair  $(A, B)$  is called a  $(N, k, M, l)$ -difference pair if  $\forall d \in \{1, \dots, M\}$ , there exists at least one ordered pair  $b_i \in B$  and  $a_j^p \in A^p$  such that  $b_i - a_j^p \equiv d \pmod{M}$ .

Consider an example where  $N = 7$  and  $M = 13$ . Let  $A = \{1, 2, 4\}$  and  $B = \{1, 3, 6, 7\}$  be two subsets of  $\{1, \dots, 7\}$  and  $\{1, \dots, 13\}$ , respectively. Then  $(A, B)$  is a  $(7, 3, 13, 4)$ -difference pair, since for  $A^2$  and  $B$ ,

$$\begin{array}{l} 1 \equiv 3 - 2 \quad 2 \equiv 6 - 4 \quad 3 \equiv 1 - 11 \quad 4 \equiv 6 - 2 \quad 5 \equiv 6 - 1 \quad 6 \equiv 7 - 1 \quad 7 \equiv 3 - 9 \\ 8 \equiv 6 - 11 \quad 9 \equiv 7 - 11 \quad 10 \equiv 1 - 4 \quad 11 \equiv 6 - 8 \quad 12 \equiv 1 - 2 \quad 13 \equiv 1 - 1 \end{array} \pmod{13}$$

**Definition 9.** *(Heterogeneous cyclic coterie pair)* Given two groups of sets  $\mathcal{X} = \{A, A + 1, \dots, A + N - 1\}$  over  $\{0, \dots, N - 1\}$  and  $\mathcal{Y} = \{B, B + 1, \dots, B + M - 1\}$  over  $\{0, \dots, M - 1\}$ , suppose  $N \leq M$  and  $p = \lceil \frac{M}{N} \rceil$ . We call  $(\mathcal{X}, \mathcal{Y})$  heterogeneous cyclic coterie pair if:  $\forall (A + i)^p \subseteq \mathcal{X}^p$  and  $(B + j) \subseteq \mathcal{Y}$ ,  $(A + i)^p \cap (B + j) \neq \emptyset$ .

**Theorem 5.** Suppose  $A = \{a_1, \dots, a_k\}$  in  $(\mathbb{Z}_N, +)$  and  $A_i = A + i$ ,  $B = \{b_1, \dots, b_k\}$  in  $(\mathbb{Z}_M, +)$  and  $B_i = B + j$ , where  $N \leq M$ . Given two groups of sets  $\mathcal{X} = \{A_i | 0 \leq i \leq N - 1\}$  and  $\mathcal{Y} = \{B_j | 0 \leq j \leq M - 1\}$ , the pair  $(\mathcal{X}, \mathcal{Y})$  is a heterogeneous cyclic coterie pair if and only if  $(A, B)$  is a  $(N, k, M, l)$ -difference pair.

*Proof. Sufficient Condition.* Without loss of generality, we assume that  $j > i$  regarding two sets  $B_i$  and  $A_j^p$ , where  $p = \lceil \frac{M}{N} \rceil$ . Consider the  $l^{\text{th}}$  element of  $B_i$  and  $m^{\text{th}}$  element of  $A_j^p$ , denoted by  $b_{i,l}$  and  $a_{j,m}^p$ , respectively. We will show that  $b_{i,l} = a_{j,m}^p$ . We have  $b_{i,l} - a_{j,m}^p = (b_l - a_m^p + i - j) \pmod{M}$ . According to the definition of  $(N, k, M, l)$ -cyclic difference pair, there must be some  $l$  and  $m$  such that  $b_l - a_m^p \equiv j - i \pmod{M}$ . Therefore, we can always choose a pair of  $l$  and  $m$  such that  $b_{i,l} - a_{j,m}^p \equiv 0 \pmod{M}$ . It implies that  $A_i^p \cap B_j \neq \emptyset$ .

*Necessary Condition.* We prove the condition by contradiction. Assume that  $A_i^p \cap B_j \neq \emptyset$  and  $(\mathcal{X}, \mathcal{Y})$  is not a  $(N, k, M, l)$ -difference pair. Then there exists a number  $t \in \{1, \dots, M - 1\}$ , say  $t$ , in which  $b_i - a_j^p \not\equiv t \pmod{M}$ ,  $\forall i, j$ . Consider the  $m^{\text{th}}$  element of  $B_0$  and the  $l^{\text{th}}$  element of  $A_i^p$ . We have  $b_{t,l} - a_{i,m}^p \equiv a_l - a_m + t \pmod{M}$ . However, we know that for some  $l$  and  $m$ ,  $b_{t,l} - a_{i,m}^p = 0$  since  $A_i^p \cap B_j \neq \emptyset$ . This implies that  $b_i - a_j^p \equiv t \pmod{M}$ , which contradicts the assumption.

If two groups of sets  $\mathcal{X}$  and  $\mathcal{Y}$  can form a *heterogeneous cyclic coterie pair*, they have at least one intersection within the larger cycle length. But the pair does not guarantee that any two sets from the same group,  $\mathcal{X}$  or  $\mathcal{Y}$ , also have an intersection.

### 3.3 Definition and Verification of Cyclic Quorum System Pair

We define the CQS-Pair as follows.

**Definition 10.** *Cyclic Quorum System Pair (CQS-Pair)*. Given two quorum sets  $\mathcal{X} = \{A, A + 1, \dots, A + N - 1\}$  over  $\{0, \dots, N - 1\}$  and  $\mathcal{Y} = \{B, B + 1, \dots, B + M - 1\}$  over  $\{0, \dots, M - 1\}$ , suppose  $N \leq M$ . We call  $(\mathcal{X}, \mathcal{Y})$  CQS-Pair if

1.  $(\mathcal{X}, \mathcal{Y})$  is a heterogeneous cyclic coterie pair; and
2.  $\mathcal{X}$  and  $\mathcal{Y}$  are cyclic quorum systems, respectively.

**Theorem 6.** Given two groups of sets  $\mathcal{X} = \{A, A + 1, \dots, A + N - 1\}$  and  $\mathcal{Y} = \{B, B + 1, \dots, B + M - 1\}$ , where  $A = \{a_1, \dots, a_k\}$  in  $(\mathbb{Z}_N, +)$  and  $B = \{b_1, \dots, b_l\}$  in  $(\mathbb{Z}_M, +)$  ( $N \leq M$ ), the pair  $(\mathcal{X}, \mathcal{Y})$  is a CQS-Pair if and only if

1.  $(A, B)$  is a  $(N, k, M, l)$ -difference pair; and

2.  $A$  is a relaxed  $(N, k)$ -difference set and  $B$  is a relaxed  $(M, l)$ -difference set.

*Proof.* If  $(A, B)$  is a  $(N, k, M, l)$ -difference pair, we have that  $(\mathcal{X}, \mathcal{Y})$  is a heterogenous cyclic coterie pair. And if  $A$  and  $B$  are relaxed difference sets respectively, we have that  $\mathcal{X}$  and  $\mathcal{Y}$  are cyclic quorum systems respectively. Similarly, we can prove that the converse is also true.

**Corollary 1.** *Given a cyclic quorum system  $\mathcal{X}$ ,  $(\mathcal{X}, \mathcal{X})$  is a CQS-Pair.*

**Theorem 7.** *Cyclic Quorum System Pair (CQS-Pair) is a solution to the h-QPS problem.*

*Proof.* According to the definition of CQS-Pair, it satisfies the heterogeneous rotation closure property. So the CQS-Pair can be a solution to the h-QPS problem according to Lemma 1.

Consider an example where  $A = \{1, 2, 4\}$  and  $\mathcal{X} = C(A, \mathbb{Z}_7)$ ,  $B = \{7, 9, 14, 15, 18\}$  and  $\mathcal{Y} = C(B, \mathbb{Z}_{21})$ . The pair  $(\mathcal{X}, \mathcal{Y})$  is a CQS-Pair. But if  $A = \{3, 5, 6\}$  and  $B = \{7, 9, 14, 15, 18\}$ , the pair  $(\mathcal{X}, \mathcal{Y})$  is NOT a CQS-Pair, although  $\mathcal{X}$  and  $\mathcal{Y}$  are cyclic quorum systems, respectively.

If  $A = \{1, 2, 4\}$  and  $\mathcal{X} = C(A, \mathbb{Z}_7)$ ,  $B = \{1, 2, 4\}$  in  $(\mathbb{Z}_{14}, +)$  and  $\mathcal{Y} = C(B, \mathbb{Z}_{14})$ ,  $(A, B)$  is a  $(7, 3, 14, 3)$ -difference pair. But  $(\mathcal{X}, \mathcal{Y})$  is NOT a CQS-Pair since  $B$  is not a relaxed difference set in  $(\mathbb{Z}_{14}, +)$  and  $\mathcal{Y}$  is not a cyclic quorum system.

### 3.4 Constructing Scheme for Cyclic Quorum System Pair

In previous work, exhaustive search has been used to find an optimal solution for cyclic quorum design [5]. This is not practical when cycle length ( $n$ ) is large. In this section, we first present a fast construction scheme for cyclic quorum systems and then apply it to the design of CQS-Pair. First, we define a few concepts.

**Definition 11. Automorphism.** *Suppose  $(X, \mathcal{A})$  is a design. An transform function  $\alpha$  is an automorphism of  $(X, \mathcal{A})$  if*

$$[\{\alpha(x) : x \in A\} : A \in \mathcal{A}] = \mathcal{A}$$

**Definition 12. Disjoint cycle representation:** *The disjoint cycle representation on a set  $X$  is a group of disjointing cycles in which each cycle has the form  $(x \ \alpha(x) \ \alpha(\alpha(x)) \ \dots)$  for some  $x \in X$ .*

Suppose the automorphism is  $x \mapsto 2x \text{ mod } 7$ . The disjoint cycle representation of  $\mathbb{Z}_7$  is as follows:  $(0) (1 \ 2 \ 4) (3 \ 6 \ 5)$ .

**Definition 13.** *Let  $D$  be a  $(v, k, \lambda)$ -difference set in  $(\mathbb{Z}_v, +)$ . For an integer  $m$ , let  $mD = \{mx : x \in D\}$ . Then  $m$  is called a multiplier of  $D$  if  $mD = D + g$  for some  $g \in \mathbb{Z}_v$ . Also, we say that  $D$  is fixed by the multiplier  $m$  if  $mD = D$ .*

**Theorem 8. (Multiplier Theorem).** *Suppose there exists a  $(v, k, \lambda)$  - difference set  $D$ . Suppose also that the following four conditions are satisfied: 1).  $p$  is prime; 2).  $\gcd(p, v) = 1$ ; 3).  $k - \lambda \equiv 0 \pmod{p}$ ; and 4).  $p > \lambda$ . Then  $p$  is a multiplier of  $D$ .*

The proof of Theorem 8 is given in [7]. According to the Theorem of Singer Difference Set, there exists a  $(q^2 + q + 1, q + 1, 1)$ -difference set when  $q$  is a prime power. So the Multiplier Theorem does not guarantee the existence of a  $(v, k, \lambda)$ -difference sets for any integer  $v$ . We only consider the  $(q^2 + q + 1, q + 1, 1)$ -design, where  $q$  is a prime power, in our construction scheme.

We first give an example to illustrate the application of the Multiplier Theorem for the construction of difference sets.

**Example.** We use the Multiplier Theorem to find a  $(21, 5, 1)$ -difference set. Observe that  $p = 2$  satisfies the conditions of Theorem 8. Hence 2 is a multiplier of any such difference set. Therefore,

the *automorphism* is  $\alpha(x) \mapsto 2x \pmod{21}$ . Thus, we get the disjoint cycle representation of the permutation defined by  $\alpha(x)$  of  $\mathbb{Z}_{21}$  as follows:

$$(0) (1\ 2\ 4\ 8\ 16\ 11) (3\ 6\ 12) (5\ 10\ 20\ 19\ 17\ 13) (7\ 14) (9\ 18\ 15)$$

The difference set we are looking for must consist of a union of cycles in the list above. Since the difference set has size five, it must be the union of one cycle of length two and one cycle of length three. There are two possible ways to do this, both of which happen to produce the difference set:

$$(3\ 6\ 7\ 12\ 14) \text{ and } (7\ 9\ 14\ 15\ 18)$$

With the Multiplier Theorem, we can quickly construct  $(q^2 + q + 1, q + 1, 1)$ -difference sets, where  $q$  is a prime power. The mechanism can significantly improve the speed of finding the optimal solution relative to the exhaustive method in [5]. After obtaining the difference sets, we use Theorem 6 to build a CQS-pair.

To check the non-empty intersection property of two heterogeneous difference sets  $A = \{a_1, a_2, \dots, a_k\}$  in  $(\mathbb{Z}_N, +)$  and  $B = \{b_1, b_2, \dots, b_l\}$  in  $(\mathbb{Z}_M, +)$  where  $N \leq M$  and  $p = \lceil \frac{M}{N} \rceil$ , let's define a  $pk \times l$  matrix  $\mathcal{M}_{l \times pk}$  whose element  $m_{i,j}$  is equal to  $(b_i - a'_j) \pmod{M}$  where  $a'_j \in A^p$ . We can check whether  $(A, B)$  is a  $(N, k, M, l)$ -difference pair by checking if  $\mathcal{M}_{l \times pk}$  contains all elements from 0 to  $M - 1$ . We call  $\mathcal{M}_{l \times pk}$  a *verification matrix*.

Since we only consider  $(q^2 + q + 1, q + 1, 1)$ -design, we describe our algorithm for constructing a CQS-Pair as follows:

- Step 1:** Given two input integers  $n, m (n \leq m)$ , find out two prime power  $q$  and  $r$  which satisfy  $n = q^2 + q + 1$  and  $m = r^2 + r + 1$ . Set  $k \leftarrow q + 1$  and  $l \leftarrow r + 1$ .
- Step 2:** Obtain the Multiplier  $p_a$  for  $(n, k, 1)$ -difference set, and  $p_b$  for  $(m, l, 1)$ -difference set; then set *automorphisms*  $\alpha_n(x) \leftarrow p_a \cdot x \pmod{n}$  and  $\alpha_m(x) \leftarrow p_b \cdot x \pmod{m}$ ;
- Step 3:** Construct the disjoint cycle presentation for  $\mathbb{Z}_n$  with  $\alpha_n(x)$ , and the disjoint cycle presentation for  $\mathbb{Z}_m$  with  $\alpha_m(x)$ , respectively;
- Step 4:** Suppose there are  $u$  unions of disjoint cycle being  $(n, k, 1)$ -difference set, and  $v$  unions of disjoint cycle being  $(m, l, 1)$ -difference set. Construct the *verification matrices* for all  $u \times v$  pairs of cyclic quorum systems  $(\mathcal{X}, \mathcal{Y})$ .
- Step 5:** For all  $u \times v$  *verification matrices*, check whether it contains all elements from 1 to  $m$ . If *true*,  $(\mathcal{X}, \mathcal{Y})$  is a CQS-Pair, otherwise, it is not a CQS-Pair.

By employing our constructing algorithm, for two different integer  $n$  and  $m$  satisfying  $n = q^2 + q + 1$  and  $m = r^2 + r + 1$  ( $q$  and  $r$  being two prime powers,  $n \leq m$ ), it will take  $O(n^2)$  and  $O(m^2)$  time for them to build the *disjoint cycle representation* respectively. After that, Step 5 in the algorithm will check  $u \times v \times l \times pk \approx uvm^{3/2}n^{-1/2}$  elements since  $l \approx \sqrt{m}$  and  $k \approx \sqrt{n}$ , where  $u$  and  $v$  are numbers of  $(n, k, 1)$ -difference sets and  $(m, l, 1)$ -difference sets, respectively. So the total time complexity is  $O(uvm^{3/2}n^{-1/2} + m^2)$  for constructing a CQS-Pair with input parameters  $n$  and  $m$  ( $n \leq m$ ).

### 3.5 A Complete Application Example

As an example, consider  $n = 7$ ,  $m = 21$ . By Multiplier Theorem, we can easily obtain two  $(7, 3, 1)$ -difference sets being  $\{1, 2, 4\}$  and  $\{3, 6, 5\}$  in  $(\mathbb{Z}_7, +)$ . Similarly, there are two  $(21, 5, 1)$ -difference sets,  $\{3, 6, 7, 12, 14\}$  and  $\{7, 9, 14, 15, 18\}$  in  $(\mathbb{Z}_{21}, +)$ . Let  $A_7 = \{1, 2, 4\}$ ,  $B_7 = \{3, 6, 5\}$ ,  $A_{21} = \{3, 6, 7, 12, 14\}$ , and  $B_{21} = \{7, 9, 14, 15, 18\}$ .



Totally, there are four pairs of  $(7, 3, 1)$ -difference sets and  $(21, 5, 1)$ -difference sets. First, we check the pair  $(C(A_7, \mathbb{Z}_7), C(A_{21}, \mathbb{Z}_{21}))$ . The constructed verification matrix is as follows.

$$\begin{bmatrix} 2 & 1 & 20 & 16 & 15 & 13 & 9 & 8 & 6 \\ 5 & 4 & 2 & 19 & 18 & 16 & 12 & 11 & 9 \\ 6 & 5 & 3 & 20 & 19 & 17 & 13 & 12 & 10 \\ 11 & 10 & 8 & 4 & 3 & 1 & 18 & 17 & 15 \\ 13 & 12 & 10 & 6 & 5 & 3 & 20 & 19 & 17 \end{bmatrix}$$

We find that 7 and 14 are not in the matrix. So the pair  $(C(A_7, \mathbb{Z}_7), C(A_{21}, \mathbb{Z}_{21}))$  is NOT a CQS-Pair. Similarly, we can check that  $(C(B_7, \mathbb{Z}_7), C(B_{21}, \mathbb{Z}_{21}))$  is NOT a CQS-Pair. But  $(C(A_7, \mathbb{Z}_7), C(B_{21}, \mathbb{Z}_{21}))$  and  $(C(B_7, \mathbb{Z}_7), C(A_{21}, \mathbb{Z}_{21}))$  are a CQS-Pair, respectively.

The CQS-Pair can be applied to sensor networks for dynamically changing the quorum system (i.e., the cycle length) in each node, in order to meet the end-to-end delay constraint without losing connectivity. Table 1 shows the available pairs for cycle lengths  $\leq 21$ .

**Table 1.** CQS-Pair (for  $n, m \leq 21$ )

cycle length	7 $A_7 = \{1, 2, 4\}$ $B_7 = \{3, 5, 6\}$	13 $A_{13} = \{0, 1, 3, 9\}$ , $B_{13} = \{0, 2, 6, 5\}$ $C_{13} = \{0, 4, 12, 10\}$ , $D_{13} = \{0, 7, 8, 11\}$	21 $A_{21} = \{3, 6, 7, 12, 14\}$ $B_{21} = \{7, 9, 14, 15, 18\}$
7	$(C(A_7, \mathbb{Z}_7), C(A_7, \mathbb{Z}_7))$ $(C(B_7, \mathbb{Z}_7), C(B_7, \mathbb{Z}_7))$	$(C(A_7, \mathbb{Z}_7), C(A_{13}, \mathbb{Z}_{13}))$ $(C(A_7, \mathbb{Z}_7), C(B_{13}, \mathbb{Z}_{13}))$ $(C(B_7, \mathbb{Z}_7), C(C_{14}, \mathbb{Z}_{13}))$ $(C(B_7, \mathbb{Z}_7), C(D_{14}, \mathbb{Z}_{13}))$	$(C(A_7, \mathbb{Z}_7), C(B_{21}, \mathbb{Z}_{21}))$ $(C(B_7, \mathbb{Z}_7), C(A_{21}, \mathbb{Z}_{21}))$
13		$(C(A_{13}, \mathbb{Z}_{13}), C(A_{13}, \mathbb{Z}_{13}))$ $(C(B_{13}, \mathbb{Z}_{13}), C(B_{13}, \mathbb{Z}_{13}))$ $(C(C_{13}, \mathbb{Z}_{13}), C(C_{13}, \mathbb{Z}_{13}))$ $(C(D_{13}, \mathbb{Z}_{13}), C(D_{13}, \mathbb{Z}_{13}))$	$(C(B_{13}, \mathbb{Z}_{13}), C(A_{21}, \mathbb{Z}_{21}))$

## 4 Performance Analysis

### 4.1 Average Delay

We denote the average delay as the time between data arrival and discovery of the adjacent receiver (two nodes wake-up simultaneously). Note that this metric does not include the time for delivering a message. And suppose the length of one time slot being 1.

**Theorem 9.** *The average delay between two nodes wakeup based on quorums from the same Cyclic Quorum System adopting the  $(n, k, 1)$ -difference set is  $E(\text{Delay}) = \frac{n-1}{2}$ .*

*Proof.* Let the  $k$  elements in  $(n, k, 1)$ -difference set be denoted as  $a_1, a_2, \dots, a_k$ . If a node has a message arrived during the  $i^{\text{th}}$  time slot, the expected delay (from data arrival to two nodes wake-up simultaneously) is  $\frac{1}{k}(a_1 - i) \bmod n + \frac{1}{k}(a_2 - i) \bmod n + \dots + \frac{1}{k}(a_k - i) \bmod n$ . If a message has arrived, the probability of the message arriving at the  $i^{\text{th}}$  time slot is  $\frac{1}{n}$ . Thus, the expected delay (average delay) is:

$$\begin{aligned} E(\text{Delay}) &= \frac{1}{n} \left[ \frac{1}{k}(a_1 - 1) \bmod n + \frac{1}{k}(a_2 - 1) \bmod n + \dots + \frac{1}{k}(a_k - 1) \bmod n \right. \\ &\quad + \frac{1}{k}(a_1 - 2) \bmod n + \frac{1}{k}(a_2 - 2) \bmod n + \dots + \frac{1}{k}(a_k - 2) \bmod n \\ &\quad + \dots \\ &\quad \left. + \frac{1}{k}(a_1 - n) \bmod n + \frac{1}{k}(a_2 - n) \bmod n + \dots + \frac{1}{k}(a_k - n) \bmod n \right] \\ &= \frac{1}{nk} \cdot (k \cdot 1 + k \cdot 2 + \dots + k \cdot n - 1) = \frac{n-1}{2} \end{aligned}$$

**Corollary 2.** *The average delay between two nodes wakeup based on a CQS-Pair in which two cyclic quorum systems have cycle length  $n$  and  $m$  ( $n \leq m$ ) respectively, is:*

$$\frac{n-1}{2} < E(\text{Delay}) < \frac{m-1}{2}$$

Corollary 2 indicates that the average delay between two nodes adopting CQS-Pair is bounded. When the average one-hop delay constraint is  $D$ , we must meet  $\frac{m-1}{2} \leq D$ .

## 4.2 Quorum Ratio and Energy Conservation

We define **quorum ratio** ( $\phi$ ) as the proportion of the beacon intervals that is required to be awake in each cycle. If the wakeup schedule in a node is a cyclic quorum system which is based on a  $(n, k, 1)$ -difference set, its quorum ratio is  $\frac{k}{n}$ . It has been proved that a  $(q^2 + q + 1, q + 1, 1)$ -difference set exists and is an optimal design for a given quorum size  $q + 1$  [12]. The optimal quorum ratio is  $\phi = \frac{q+1}{q^2+q+1}$  for such a cyclic quorum system.

For a CQS-Pair, the quorum ratios for systems in the pair which are based on  $(N, k, M, l)$ -difference pair are  $\phi_1 = \frac{\sqrt{4N-3}-1}{2N}$  and  $\phi_2 = \frac{\sqrt{4M-3}-1}{2M}$  respectively, when we only consider  $(q^2 + q + 1, q + 1, 1)$ -design in the CQS-Pair constructing scheme.

Note that a  $\sqrt{n} \times \sqrt{n}$  grid quorum system also satisfies the heterogeneous rotation closure property and can be applied as a solution to the h-QPS problem. The quorum ratio is  $\phi = \frac{2\sqrt{n}-1}{n}$  for a grid quorum system.

The energy conservation ratio is correlated with the quorum ratio. If the quorum ratio is  $\phi$ , the energy conservation ratio for a node is  $1 - \phi$ .

## 4.3 Multicast/Broadcast Support

The quorum-based asynchronous wakeup protocols cannot guarantee that more than one receiver is awake when the transmitter requests to transmit a multicast/broadcast message.

There are multiple ways to support multicast/broadcast. One method is to adopt relatively prime frequencies among all nodes for wakeup scheduling. This method does not need synchronization between the transmitter and all receivers. The transmitter only needs to notify  $m$  receivers to wake up via the pairwise relative primes  $p_1, p_2, \dots, p_m$ , respectively. Then each receiver generates its new wakeup frequency based on the received frequency. Through Chinese Remainder Theorem, it can be proven that the  $m$  receivers must wakeup simultaneously at the  $I^{\text{th}}$  beacon interval ( $0 \leq I \leq p_1 \times p_2 \dots \times p_m$ ). The transmitter can then transmit a multicast/broadcast message at this interval.

Another way to multicast/broadcast is by using synchronization over quorum-based wakeup schedule. The transmitter can book-keep all neighbors' schedules, and synchronize their schedules so that neighboring nodes wake up in the same set of slots with the use of Lamport's clock synchronization algorithm [4]. When all nodes are awake simultaneously, the transmitters then send a message to multiple neighbors simultaneously.

The first mechanism has the advantage that no synchronization is needed between transmitter and multiple receivers. But it cannot bound the average delay. The second approach can bound the average delay but it needs bookkeeping and synchronization over asynchronous wakeup schedules.

## 5 Past and Related Work

As mentioned in the Section 1, there are three categories of wakeup mechanism for wireless sensor networks. We summarize them and overview past and related work in asynchronous wakeup scheduling.

**On-Demand Wakeup.** The implementation of on-demand wakeup schemes typically requires two different channels: a data channel and a wakeup channel for awaking nodes when needed. This allows for not deferring the transmission of signal on the wakeup channel if a packet transmission is in progress on the other channel, thus reducing the wakeup latency. The drawback is the additional cost for the second radio. STEM (Sparse Topology and Energy Management) [6] uses two different radios for wakeup signals and data packet transmissions, respectively. The key idea is that a node remains awake until it has not received any message destined for it for a certain time. STEM uses separate control and data channels, and hence the contention among control and data messages is alleviated. The energy efficiency of STEM is dependent on that of the control channel.

**Scheduled Rendezvous Schemes.** These schemes require that all neighboring nodes wake up at the same time. Different scheduled rendezvous protocols differ in the way network nodes sleep and wakeup during their lifetime. The simplest way is using a Fully Synchronized Pattern, like S-MAC [11]. In this case, all nodes in the network wakeup at the same time according to a periodic pattern. A further improvement can be achieved by allowing nodes to switch off their radio when no activity is detected for at least a timeout value, like that in T-MAC [1]. The disadvantages are the complexity and overhead for synchronization.

**Asynchronous wakeup.** This was first introduced in [10] with reference to IEEE 802.11 ad hoc networks. The authors proposed three different asynchronous sleep/wakeup schemes that require some modifications to the basic IEEE 802.11 Power Saving Mode (PSM). More recently, Zheng et al. [12] took a systematic approach to design asynchronous wakeup mechanisms for ad hoc networks (applicable for wireless sensor networks as well). They formulate the problem of generating wakeup schedules as a block design problem and derive theoretical bounds under different communication models. The basic idea is that each node is associated with a Wakeup Schedule Function (WSF) that is used to generate a wakeup schedule. For two neighboring nodes to communicate, their wakeup schedules have to overlap regardless of the difference in their clocks.

For the quorum-based asynchronous wakeup design, Luk and Wong [5] designed a cyclic quorum system using difference sets. But they do exhaustive search to obtain a solution for each cycle length  $N$ , which is impractical when  $N$  is large.

**Asymmetric quorum design.** In the clustered environment of sensor networks, it is not always necessary to guarantee all-pair neighbor discovery. The Asymmetric Cyclic Quorum (ACQ) system [9] was proposed to guarantee neighbor discovery between each member node and the clusterhead, and between clusterheads in a network. The authors also presented a construction scheme which assembles the ACQ system in  $O(1)$  time to avoid exhaustive searching. ACQ is a generalization of the cyclic quorum system. The scheme is configurable for different networks to achieve different distribution of energy consumption between member nodes and the clusterhead.

However, it remains a challenging issue to efficiently design an asymmetric quorum system given an arbitrary value of  $n$ . One previous study [12] shows that the problem of finding an optimal asymmetric block design can be reduced to the minimum vertex cover problem, which is NP-complete. However, the ACQ [9] construction is not optimal in that the quorum ratio for symmetric-quorum is  $\phi = \lceil \frac{n+1}{2} \rceil$  and the quorum ratio for asymmetric-quorum is  $\phi' = \lceil \sqrt{\frac{n+1}{2}} \rceil$ . The another drawback is that it cannot be a solution to the h-QPS problem since the two asymmetric-quorums cannot guarantee the intersection property.

**Transport layer approach.** Wang et al. [8] applied quorum based wakeup schedule at the transport layer which can cooperate with any MAC layer protocol, allowing for the reuse of well-understood MAC protocols. The proposed technique saves idle energy by relaxing the requirement for end-to-end connectivity during data transmission and allowing the network to be disconnected intermittently via scheduled sleeping. The limitation of this work is that each node adopts same grid quorum systems as wakeup scheduling and the quorum ratio is not optimal comparing with that of cyclic quorum systems.

## 6 Conclusions

This paper presents a theoretical approach for heterogeneous asynchronous wakeup scheduling in wireless sensor networks. We defined the h-QPS problem—i.e., given two cycle lengths  $n$  and  $m$  ( $n < m$ ), how to design a pair of heterogeneous quorum systems to guarantee that two adjacent nodes picking up heterogeneous quorums from the pair as their wakeup schedule can hear each other at least once in every  $m$  consecutive time slots. We defined the Cyclic Quorum System Pair (CQS-Pair) which can be applied as a solution to h-QPS problem. We also presented a fast construction scheme to assemble a CQS-Pair. In our construction scheme, we first quickly construct  $(n, k, 1)$ -difference set and  $(m, l, 1)$ -difference set. Based two difference sets  $A$  in  $(\mathbb{Z}_n, +)$  and  $B$  in  $(\mathbb{Z}_m, +)$ , we can construct a CQS-Pair  $(C(A, \mathbb{Z}_n), C(B, \mathbb{Z}_m))$  when  $A$  and  $B$  can form a  $(n, k, m, l)$ -difference pair.

The performance of a CQS-Pair was analyzed in terms of average delay, quorum ratio, and issues for supporting multicast/broadcast. We show that the average delay between two node wakeup via heterogeneous quorums from a CQS-Pair is bounded between  $\frac{n-1}{2}$  and  $\frac{m-1}{2}$ , and the quorum ratios of the two quorum systems in the pair are optimal respectively given their cycle lengths  $n$  and  $m$ .

There are several directions for future work. One direction is to find a scheme to check the existence of a CQS-Pair for any given  $n$  and  $m$ . Another example is to extend the CQS-Pair to CQS  $m$ -pair in which  $m$  cyclic quorum systems have the heterogeneous rotation closure property with one another.

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