FBLT: A Real-Time Contention Manager with Improved Schedulability

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Abstract—We consider software transactional memory (STM) concurrency control for embedded multicore real-time software, and present a novel contention manager for resolving transactional conflicts, called FBLT. We upper bound transactional retries and task response times under FBLT, and identify when FBLT has better real-time schedulability than the previous best contention manager, PNF. Our implementation in the Rochester STM framework reveals that FBLT yields shorter or comparable retry costs than competitor methods.

I. INTRODUCTION

Embedded systems sense physical processes and control their behavior, typically through feedback loops. Since physical processes are concurrent, computations that control them must also be concurrent, enabling them to process multiple streams of sensor input and control multiple actuators, all concurrently while satisfying time constraints.

The de facto standard for concurrent programming is the threads abstraction, and the de facto synchronization abstraction is locks. Lock-based concurrency control has significant programmability, scalability, and composability advantages [1]. Transactional memory (TM) is an alternative synchronization model for shared memory objects that promises to alleviate these difficulties. With TM, code that read/write shared objects is organized as memory transactions, which execute speculatively, while logging changes made to objects. Two transactions conflict if they access the same object and at least one access is a write. When that happens, a contention manager (CM) [2] resolves the conflict by aborting one and allowing the other to commit, yielding (the illusion of) atomicity. Aborted transactions are re-started, after rolling back the changes. In addition to a simple programming model, TM provides performance comparable to lock-free approach, especially for high contention and read-dominated workloads (see an example TM system's performance in [3]), and is composable [4]. TM has been proposed in hardware, called HTM, and in software, called STM, with the usual tradeoffs: HTM has lesser overhead, but needs transactional support in hardware; STM is available on any hardware.

Given STM’s programmability, scalability, and composability advantages, it is a compelling concurrency control technique also for multicore embedded real-time software. However, this requires bounding transactional retries, as real-time threads, which subsume transactions, must satisfy time constraints. Retry bounds under STM are dependent on the CM policy at hand.

Past real-time CM research has proposed resolving transactional contention using dynamic and fixed priorities of parent threads, resulting in Earliest Deadline CM (ECM) and Rate Monotonic CM (RCM) [5]–[7], which are intended to be used with global EDF (G-EDF) and global RMS (G-RMS) multicore real-time schedulers [8], respectively. In particular, [6] shows that ECM and RCM achieve higher schedulability – i.e., greater number of task sets meeting their time constraints – than lock-free synchronization only under some ranges for the maximum atomic section length. That range is significantly expanded with the Length-based CM (LCM) in [7], increasing the coverage of STM’s timeliness superiority. ECM, RCM, and LCM suffer from transitive retry (Section III) and cannot handle multiple objects per transaction efficiently. These limitations are overcome with the Priority with Negative value First access CM (PNF) [9], [10]. However, PNF requires a-priori knowledge of all objects accessed by each transaction. This significantly limits programmability, and is incompatible with dynamic STM implementations [11]. Additionally, PNF is a centralized CM, which increases overheads and retry costs, and has a complex implementation.

We propose the First Bounded, Last Timestamp CM (or FBLT) (Section IV). In contrast to PNF, FBLT does not require a-priori knowledge of objects accessed by transactions. Moreover, FBLT allows each transaction to access multiple objects with shorter transitive retry cost than ECM, RCM and LCM. Additionally, FBLT is a decentralized CM and does not use locks in its implementation. Implementation of FBLT is also simpler than PNF. We establish FBLT’s retry and response time upper bounds under G-EDF and G-RMS schedulers (Section V). We also identify the conditions under which FBLT’s schedulability is better than PNF (Section VI). We implement FBLT and competitor CM techniques in the Rochester STM framework [12] and conduct experimental studies (Section VII). Our results reveal that FBLT has shorter retry cost than ECM, RCM, LCM and lock-free. FBLT’s retry cost is comparable to that of PNF, especially in case of non-transitive retry, but it doesn’t require a-priori knowledge of objects accessed by transactions, unlike PNF.

Thus, the paper’s contribution is the FBLT contention manager with superior timeliness properties. FBLT, thus allows programmers to reap STM’s significant programmability and composability benefits for a broader range of multicore embedded real-time software than what was previously possible.
II. Preliminaries

We consider a multiprocessor system with $m$ identical processors and $n$ sporadic tasks $t_1, t_2, \ldots, t_n$. The $k^{th}$ instance (or job) of a task $t_i$ is denoted $t_i^k$. Each task $t_i$ is specified by its worst case execution time (WCET) $c_i$, its minimum period $T_i$ between any two consecutive instances, and its relative deadline $D_i$, where $D_i = T_i$. Job $t_i^k$ is released at time $r_i^k$ and must finish no later than its absolute deadline $d_i^k = r_i^k + D_i$. Under a fixed priority scheduler such as G-RMA, $p_i$ determines $t_i$'s (fixed) priority and it is constant for all instances of $t_i$. Under a dynamic priority scheduler such as G-EDF, a job $t_i^k$'s priority, $p_i^k$, differs from one instance to another. A task $t_j$ may interfere with task $t_i$ for a number of times during an interval $L$, and this number is denoted as $G_{ij}(L)$.

Shared objects. A task may need to read/write shared, in-memory data objects while it is executing any of its atomic sections (transactions), which are synchronized using STM. The set of atomic sections of task $t_i$ is denoted $s_i$. $s_i^k$ is the $k^{th}$ atomic section of $t_i$. Each object, $\Theta$, can be accessed by multiple tasks. The set of distinct objects accessed by $t_i$ is $\Theta_i$, without repeating objects. The set of atomic sections used by $t_i$ to access $\Theta$ is $s_i(\Theta)$. The set of lengths of those atomic sections is $\text{len}(s_i(\Theta))$. $s_i^k(\Theta)$ is the $k^{th}$ atomic section of $t_i$ that accesses $\Theta$, $s_i^k$ can access one or more objects in $\Theta$. So, $s_i^k$ refers to the transaction itself, regardless of the objects accessed by the transaction. We denote the set of all accessed objects by $s_i^k \Theta^k$. While $s_i^k(\Theta)$ implies that $s_i^k$ accesses an object $\theta \in \Theta^k, s_i^k(\Theta)$ implies that $s_i^k$ accesses a set of objects $\Theta = \{ \theta \in \Theta^k \}$. $s_i^k = s_i^k(\Theta^k)$ refers only once to $s_i^k$, regardless of the number of objects in $\Theta$. So, $\{ s_i^k(\Theta) \} \setminus \Theta_i = 1$. $s_i^k(\Theta)$ executes for a duration $\text{len}(s_i^k(\Theta))$. $\text{len}(s_i^k(\Theta)) = \text{len}(s_i^k(\Theta)) = \text{len}(s_i^k(\Theta^k))$. The set of tasks sharing $\Theta$ with $t_i$ is denoted $\gamma_i(\Theta)$.

Atomic sections are non-nested (supported nested STM is future work). The maximum-length atomic section in $\gamma_i$ that accesses $\Theta$ is denoted $s_{\gamma_i}(\Theta)$, while the maximum one among all tasks is $s_{\gamma_{\max}}(\Theta)$, and the maximum one among tasks with priorities lower than that of $t_i$ is $s_{\gamma_{\max}}(\Theta) = \max\{ s_{\gamma_{\max}}(\Theta) : \Theta \in \Theta_i \}$.

STM retry cost. If two or more atomic sections conflict, the CM will commit one section and abort and retry the others, increasing the time to execute the aborted sections. The increased time that an atomic section $s_i^j(\Theta)$ will take to execute due to a conflict with another section $s_i^k(\Theta)$, is denoted $W_i^j(s_i^k(\Theta))$. If an atomic section, $s_i^k$, is already executing, and the transaction that tries to access a shared object with $s_i^k$, then $s_i^k$ is said to “interfere” or “conflict” with $s_i^k$. The job $s_i^k$ is the “interfering job”, and the job $s_i^k$ is the “interfered job”.

Due to transitive retry (introduced in Section III), an atomic section $s_i^k(\Theta)$ may retry due to another atomic section $s_i^l(\Theta)$, where $\Theta_k^l \cap \Theta_k^j = \emptyset$. $\Theta_k$ denotes the set of objects not accessed directly by atomic sections in $t_i$, but can cause transactions in $t_i$ to retry due to transitive retry. $\theta_i^{\gamma_i} = (\Theta_i) \setminus \Theta_i$ is the set of all objects that can cause transactions in $t_i$ to retry directly or through transitive retry. $\gamma_i^{\gamma_i}$ is the set of tasks that accesses objects in $\Theta_i$. $\gamma_i^{\gamma_i}(\Theta_i) = (\gamma_i + \gamma_i)$ is the set of all tasks that can directly or indirectly (through transitive retry) cause transactions in $t_i$ to abort and retry.

The total time that a task $t_i$’s atomic sections have to retry over $T_i$ is denoted $RC(T_i)$. The additional amount of time by which all interfering jobs of $t_i$ increases the response time of any job of $t_i$ during $L$, without considering retries due to atomic sections, is denoted $W_{ij}(L)$.

III. Motivation

ECM [6], RCM [6], and LCM [7] suffer from transitive retry. Transitive retry is illustrated by the following example:

Consider three atomic sections $s_i^1$, $s_i^2$, and $s_i^3$ belonging to jobs $t_i^1$, $t_i^2$, and $t_i^3$, with priorities $p_1^2 > p_2^3 > p_3^1$, respectively. Assume that $s_i^1$ and $s_i^2$ share objects, and $s_i^2$ and $s_i^3$ share objects. $s_i^1$ and $s_i^3$ do not share objects. Now, $s_i^2$ can cause $s_i^3$ to retry, which in turn will cause $s_i^1$ to retry. This means that $s_i^1$ will retry transitorily because of $s_i^3$, which will increase the retry cost of $s_i^1$. Now, consider another atomic section $s_i^4$ with a priority higher than that of $s_i^3$. Suppose $s_i^4$ shares objects only with $s_i^3$. Thus, $s_i^4$ can cause $s_i^3$ to retry, which in turn will cause $s_i^2$ to retry, and finally, $s_i^3$ to retry. Thus, transitive retry will move from $s_i^2$ to $s_i^1$, increasing the retry cost of $s_i^1$. The situation gets worse as more higher priority tasks are added, where each task shares objects with its immediate lower priority task. $t_i^1$ may have atomic sections that share objects with $t_i^2$, but this will not prevent the effect of transitive retry due to $s_i^1$.

Definition 1: Transitive retry. A transaction $s_i^k$ suffers from transitive retry when $s_i^k$ retries due to a higher priority transaction $s_i^h$, and $\Theta_i^k \cap \Theta_i^h = \emptyset$.

Therefore, the analysis in [6] and [7] extends the set of objects that can cause an atomic section of a lower priority job to retry. This is done by initializing the set of conflicting objects, $\gamma_i$, to all objects accessed by all transactions of $t_i$. We then cycle through all transactions belonging to all other higher priority tasks. Each transaction $s_i^l$ that accesses at least one of the objects in $\gamma_i$ adds all other objects accessed by $s_i^l$ to $\gamma_i$. The loop over all higher priority tasks is repeated, each time with the new $\gamma_i$, until there are no more transactions accessing any object in $\gamma_i$. The final set of objects (tasks) that can cause transactions in $t_i$ to retry is $\Theta_i^{\gamma_i}(\gamma_i^{\gamma_i})$, respectively.

PFN [9], [10] is designed to avoid transitive retry by concurrently executing at most $m$ non-conflicting transactions together. These executing transactions are non-preemptive. Thus, executing transactions cannot be aborted due to direct or indirect conflict with other transactions. However, with PNF, all objects accessed by each transaction must be known a-priori. Therefore, this is not suitable with dynamic STM implementations [11]. Additionally, PNF is implemented in [10] as a centralized CM that uses locks. This increases overhead.

Thus, we propose the First-Bounded, Last Timestamp contention manager (or FBLT) that achieves the following goals:

1) reduce the retry cost of each transaction $s_i^k$ due to another transaction $s_i^l$, just as LCM [7] does compared to ECM [6] and RCM [6].

\footnote{However, note that, this solution may over-extend the set of conflicting objects, and may even contain all objects accessed by all tasks.}
2) avoid or bound the effect of transitive retries, similar to PNF [9], [10], without prior knowledge of accessed objects by each transaction, enabling dynamic STM.
3) decentralized design and avoid the use of locks, thereby reducing overhead.

IV. THE FBLT CONTENTION MANAGER

**ALGORITHM 1: The FBLT Algorithm**

```
Data: s_i^k: interfered transaction; s_j^k: interfering transactions; 
\delta^k_i: the maximum number of times s_i^k can be aborted during T_i; 
\eta^k_i: number of times s_i^k has already been aborted up to now; 
m_set: contains at most m non-preemptive transactions. m is number of processors; 
m_prio: priority of any transaction in m_set. m_prio is higher than any priority of any real-time task; 
r(s_k): time point at which s_k joined m_set; 
Result: atomic sections that will abort
1 if s_i^k, s_j^k \notin m_set then 
2 \quad \text{Apply LCM [7];}
3 \quad \text{if } s_i^k \text{ is aborted then}
4 \quad \quad \text{if } \eta^k_i \leq \delta^k_i \text{ then}
5 \quad \quad \quad \text{Increment } \eta^k_i \text{ by 1;}
6 \quad \quad \quad \text{Add } s_i^k \text{ to } m\text{-set;}
7 \quad \quad \quad \text{Record } r(s_i^k);
8 \quad \quad \quad \text{Increase priority of } s_i^k \text{ to } m\_prio;
9 \quad \quad \text{end}
10 \quad \text{else}
11 \quad \quad \text{Swap } s_i^k \text{ and } s_j^k;
12 \quad \quad \text{Go to Step 3;}
13 \quad \text{end}
14 \text{end}
15 \text{else if } s_i^k \in m\_set, s_j^k \notin m\_set then
16 \quad \text{Abort } s_j^k;
17 \quad \text{if } \eta^k_i < \delta^k_i \text{ then}
18 \quad \quad \text{Increment } \eta^k_i \text{ by 1;}
19 \quad \text{else}
20 \quad \quad \text{Add } s_i^k \text{ to } m\_set;
21 \quad \quad \text{Record } r(s_i^k);
22 \quad \quad \text{Increase priority of } s_i^k \text{ to } m\_prio;
23 \quad \text{end}
24 \text{end}
25 \text{else if } s_i^k \in m\_set, s_j^k \notin m\_set then
26 \quad \text{Swap } s_i^k \text{ and } s_j^k;
27 \quad \text{Go to Step 15;}
28 \text{end}
29 \text{else}
30 \quad \text{if } r(s_i^k) < r(s_j^k) \text{ then}
31 \quad \quad \text{Abort } s_i^k;
32 \quad \text{else}
33 \quad \quad \text{Abort } s_j^k;
34 \text{end}
35 end
```

Algorithm 1 illustrates FBLT. Each transaction \( s_i^k \) can be aborted during \( T_i \) for at most \( \delta^k_i \) times. \( \eta^k_i \) records the number of times \( s_i^k \) has already been aborted up to now. If \( s_i^k \) and \( s_j^k \) have not joined the \( m\_set \) yet, then they are preemptive transactions. Preemptive transactions resolve conflicts using LCM [7] (step 2). Thus, FBLT defaults to LCM when no transaction reaches its \( \delta \). If only one of the transactions is in the \( m\_set \), then the non-preemptive transaction (the one in \( m\_set \)) aborts the other one (steps 15 to 26). \( \eta^k_i \) is incremented each time \( s_i^k \) is aborted as long as \( \eta^k_i < \delta^k_i \) (steps 5 and 18). Otherwise, \( s_i^k \) is added to the \( m\_set \) and its priority is increased to \( m\_prio \) (steps 7 to 9 and 20 to 22). When the priority of \( s_i^k \) is increased to \( m\_prio \), \( s_i^k \) becomes a non-preemptive transaction. Non-preemptive transactions cannot be aborted by other preemptive transactions, nor by any other real-time job. The \( m\_set \) can hold at most \( m \) concurrent transactions. If there are \( m \) processors in the system, \( r(s_i^k) \) records the time \( s_i^k \) joined the \( m\_set \) (steps 8 and 21). When non-preemptive transactions conflict together (step 27), the transaction with the smaller \( r() \) commits first (steps 29 and 31). Thus, non-preemptive transactions are executed in FIFO order of the \( m\_set \).

V. RETRY COST AND RESPONSE TIME BOUNDS

We now derive an upper bound on the retry cost of any job \( \tau^+_i \) under FBLT during an interval \( L \leq T_i \). Since all tasks are sporadic (i.e., each task \( \tau_i \) has a minimum period \( T_i \)), \( T_i \) is the maximum study interval for each task \( \tau_i \).

**Claim 1:** The total retry cost for any job \( \tau^+_i \) under FBLT due to 1) conflicts between its transactions and transactions of other jobs during an interval \( L \leq T_i \) and 2) release of higher priority jobs is upper bounded by:

\[
RC_{\tau_i}(L) \leq \sum_{s_i^k \in x_i} \left( \delta^k_i \text{len}(s_i^k) + \sum_{s_j^k \in x_i^k} \text{len}(s_j^k) \right) + RC_{re}(L)
\]

where \( \chi^k_i \) is the set of at most \( m - 1 \) maximum length transactions conflicting directly or indirectly (through transitive retry) with \( s_i^k \). Each transaction \( s_i^k \) \( \in \chi^k_i \) belongs to a distinct task \( \tau_i \). \( RC_{re}(L) \) is the retry cost resulting from the release of higher priority jobs which preempt \( \tau^+_i \). \( RC_{re}(L) \) is calculated by (6.8) in [10] for G-EDF, and (6.10) in [10] for G-RMA schedulers.

**Proof:** By the definition of FBLT, \( s_i^k \in \chi^k_i \) can be aborted a maximum of \( \delta^k_i \) times before \( s_i^k \) joins the \( m\_set \). Before joining the \( m\_set \), \( s_i^k \) can be aborted due to higher priority transactions, or transactions in the \( m\_set \). The original priority of transactions in the \( m\_set \) can be higher or lower than \( \eta^k_i \). Thus, the maximum time \( s_i^k \) is aborted before joining the \( m\_set \) occurs if \( s_i^k \) is aborted for \( \delta^k_i \) times.

Transactions preceding \( s_i^k \) in the \( m\_set \) can conflict directly with \( s_i^k \), or indirectly through transitive retry. The worst case scenario for \( s_i^k \) after joining the \( m\_set \) occurs if \( s_i^k \) is preceded by \( m - 1 \) maximum length conflicting transactions. Hence, in the worst case, \( s_i^k \) has to wait for the previous \( m - 1 \) transactions to commit first. The priority of \( s_i^k \) after joining the \( m\_set \) is higher than any real-time job. Therefore, \( s_i^k \) is not aborted by any job. If \( s_i^k \) has not joined the \( m\_set \) yet, and a higher priority job \( \tau^+_j \) is released while \( s_i^k \) is running, then \( s_i^k \) may be aborted if \( \tau^+_j \) has conflicting transactions with \( s_i^k \). \( \tau^+_j \) causes only one abort in \( \tau_i^+ \) because \( \tau^+_j \) preempts \( \tau^+_i \) only once. If \( s_i^k \) has already joined the \( m\_set \), then \( s_i^k \) cannot be aborted by the release of higher priority jobs. Thus, the maximum number of times transactions in \( \tau_i^+ \) can be aborted due to the release of higher priority jobs is less than or equal to the number of interfering higher priority jobs to \( \tau_i^+ \). Claim follows.

**Claim 2:** Under FBLT, the blocking time of a job \( \tau_i^+ \) due
to lower priority jobs is upper bounded by:

\[
D(\tau^p_i) = \sum \left( \max_{m} \left( s_{j_{\text{max}}}, \forall j^l, p^l_j < p^p_i \right) \right)
\]

(2)

where \( s_{j_{\text{max}}} \) is the maximum length transaction in any job \( \tau^l \) with original priority lower than \( p^p_i \). The right hand side of (2) is the sum of the \( m \) maximum transactional lengths in all jobs with lower priority than \( \tau^p_i \).

**Proof:** The worst case blocking time for \( \tau^p_i \) occurs when the maximum length \( m \) transactions in lower priority jobs than \( \tau^p_i \) are executing non-preemptively. After commit of each transaction in the \( m \) set, a higher priority job \( \tau^p_j \) than \( \tau^p_i \) is released. So, \( \tau^p_j \) allocates the released processor instead of \( \tau^p_i \). Consequently, \( \tau^p_i \) has to wait for the whole maximum length \( m \) transactions of lower priority jobs. Claim follows.

**Claim 3:** The response time of any job \( \tau^p_i \) during an interval \( L \leq T_i \) under FBLT is upper bounded by:

\[
R^{up}_i = c_i + RC_{to}(L) + D(\tau^p_i) + \left[ \frac{1}{m} \sum_{j \neq i} W_{ij}(R^{up}_j) \right]
\]

(3)

where \( RC_{to}(L) \) is calculated by (1), \( D(\tau^p_i) \) is calculated by (2), and \( W_{ij}(R^{up}_j) \) is calculated by (11) in [6] for G-EDF, and (17) in [6] for G-RMA schedulers. (11) and (17) in [6] inflates \( c_j \) of any job \( \tau^j_i \neq \tau^p_i, p^j_i > p^p_i \) by the retry cost of transactions in \( \tau^p_i \).

**Proof:** The response time of a job is calculated directly from FBLT’s behavior. The response time of any job \( \tau^p_i \) is the sum of its worst case execution time \( c_i \), plus the retry cost of transactions in \( \tau^p_j \) (\( RC_{to}(L) \)), plus the blocking time of \( \tau^p_i \) (\( D(\tau^p_i) \)), and the workload interference of higher priority jobs. The workload interference of higher priority jobs scheduled by G-EDF is calculated by (11) in [6], and by (17) in [6] for G-RMA. Claim follows.

### VI. Schedulability Comparison

We now (formally) compare the schedulability of FBLT against PNF [9], [10]. Toward this, we compare the total utilization under FBLT with that under PNF. In this comparison, we use the inflated execution time of the task, which is the sum of the worst-case execution time of the task and its retry cost, in the utilization calculation of the task.

Let \( RC_A(T_i) \) and \( RC_B(T_i) \) denote the retry cost of a job \( \tau^p_i \) during \( T_i \) using the synchronization methods \( A \) and \( B \), respectively. Let \( D_A(\tau_i) \) and \( D_B(\tau_i) \) be the maximum blocking time of any job \( \tau^p_i \) due to lower priority jobs by methods \( A \) and \( B \) respectively. Now, schedulability of \( A \) is comparable to \( B \) if:

\[
\sum_{\forall \tau_i} \frac{c_i + RC_A(T_i) + D_A(\tau_i)}{T_i} \leq \sum_{\forall \tau_i} \frac{c_i + RC_B(T_i) + D_B(\tau_i)}{T_i}
\]

(4)

**Claim 4:** Schedulability of FBLT is equal or better than PNF if: 1) For each transaction \( s^k_j \), maximum abort times \( \delta^k \) equals at most the ratio between difference of total length of all transactions that can conflict only with \( s^k_i \) and total length of at most \( m - 1 \) longest transactions that can conflict directly or transitively with \( s^k_i \). 2) For any job \( \tau^p_i \), ratio between longest transaction in \( \tau^p_i \) or lower priority jobs to smallest transaction in lower priority jobs equals at most the ratio between minimum number of times \( \tau^p_i \) can be blocked due to non-conflicting transactions in all lower priority jobs to maximum release time of all jobs not belonging to \( \tau_i \).

**Proof:**

Substitute \( RC_A(T_i) \) and \( RC_B(T_i) \) in (4) with (1) and (3) in [9] respectively. Let \( D_A(\tau_i) \) and \( D_B(\tau_i) \) by (2) and (4) in [9] respectively. Substituting \( RC_{re}(T_i) = \sum_{\forall \tau_j \in C_i} \left( \left( \frac{T_{j}}{T_{i}} \right) + 1 \right) s_{i_{\text{max}}} \), covers \( RC_{re}(T_i) \) given by (6.8) and (6.10) in [10] and maintains correctness of (4). \( \text{zeta}_i \) is the set of higher priority tasks than any job of \( \tau_i \).

Let \( \beta_1 = \sum_{\forall s^k_j} \left( \delta^k \cdot \text{len}(s^k_j) + \sum_{s^k_i \in X^k} \text{len}(s^k_i) \right) \), \( \beta_2 = \sum_{\forall \tau_j \in C_i} \left( \left( \frac{T_{j}}{T_{i}} \right) + 1 \right) \sum_{s^k_j} \text{len}(s^k_j) \), and \( \beta_3 = \left[ \frac{1}{m} \sum_{\forall \tau_j \in C_i} \left( \left( \frac{T_{j}}{T_{i}} \right) + 1 \right) \sum_{s^k_j} \text{len}(s^k_j) \right] \). So, (4) holds if:

\[
\sum_{\forall \tau_j} \frac{\beta_1}{T_j} \leq \sum_{\forall \tau_j} \frac{\beta_2}{T_j} \leq \sum_{\forall \tau_j} \frac{\beta_3}{T_j}
\]

(5)

or \( \forall \tau_i \)

\[
\beta_1 \leq \beta_2 \quad \text{and} \quad \beta_2 \leq \beta_3
\]

According to first part of (7)

\[
\sum_{\forall s^k_j} \left( \delta^k \cdot \text{len}(s^k_j) + \sum_{s^k_i \in X^k} \text{len}(s^k_i) \right) \leq \sum_{\forall \tau_j \in C_i} \left( \left( \frac{T_{j}}{T_{i}} \right) + 1 \right) \sum_{s^k_j} \text{len}(s^k_j)
\]

(8)

For each \( s^k_j \) in \( s_i \), there are a set of zero or more \( s^k_j(\Theta) \) in \( \tau_j \) that are conflicting with \( s^k_i \). Assuming this set of conflicting transactions with \( s^k_i \) is denoted as \( \eta^k_i(\Theta) = \{ s^k_j(\Theta) \in \tau_j : (\Theta \in \theta_i \wedge \tau_j \neq \tau_i) \wedge (s^k_j(\Theta) \notin s_i^k, l \neq k) \} \). The last condition \( s^k_j(\Theta) \notin s_i^k \) ensures that common transactions \( s^k_j \) that can conflict with more than one transaction \( s_i^k \in \tau_i \) are split among different \( \eta^k_i(\Theta) \), \( k = 1, \ldots, |s_i| \). This condition is necessary because in PNF, no two or more transactions of \( \tau^p_i \) can be aborted by the same transaction of \( \tau^p_j \). Let \( \gamma_i \) be subset of \( \gamma_i \) that contains tasks with transactions conflicting directly with \( s^k_i \). By substitution of \( \eta^k_i(\Theta) \) and \( \gamma_i \) in (8), (9) holds if \( \forall s^k_i \):

\[
\sum_{\forall \tau_j \in C_i} \left( \left( \frac{T_{j}}{T_{i}} \right) + 1 \right) \sum_{s^k_j(\Theta) \in \tau_j} \frac{\text{len}(s^k_i)}{\text{len}(s^k_i)}
\]

(9)

By definition of \( \eta^k_i(\Theta) \), if \( s^k_j(\Theta) \) can conflict with \( s^k_i \) and \( s_i^k \), then \( s^k_j(\Theta) \) belongs either to \( \eta^k_i \) or \( \eta^k_i \), but not both. Let
\[ \tilde{\eta}^k(i) = \eta^k(i) - \left\{ s^h_j(\theta) | s^h_j(\theta) \text{ can belong to } \eta^l, l \neq k \right\}. \]

So, \( \tilde{\eta}^k(i) \) equals \( \eta^k(i) \) excluding any transaction that can belong to another \( \eta^l(i), l \neq k \). (9) holds if

\[
\delta^k_i \leq \frac{\sum_{\forall \tau_j \in \gamma^k_i} \left( \left\lceil \frac{T^j}{T^i} \right\rceil + 1 \right) \sum_{\forall \chi_j(\theta) \in \tilde{\eta}^k(i)} \text{len}(s^h_j(\theta))}{\text{len}(s_i^k)} - \frac{\sum_{\forall \chi_j \in \chi^k_i} \text{len}(s^h_j)}{\text{len}(s_i^k)} \]

(10)

Now, we consider the second part of (7). Let \( s_{i,j_{\text{max}}} = \max_{j \in \gamma_i} (s_{i,j}, s_{j_{\text{max}}}) \). So, \( s_{i,j_{\text{max}}} \) is the maximum transactional length in any job of \( \tau_i \) or any lower priority job. Let \( s_{j_{\text{min}}} = \min \left\{ \text{len}(s^h_j), \forall s^h_j \in \gamma_i \right\} \). So, \( s_{j_{\text{min}}} \) is the smallest transactional length in any job of \( \tau_j \) with lower priority than any job of \( \tau_i \). ∴ \( \sum_{\max_{j \in \gamma_i}} \left\{ s_{j_{\text{max}}} \right\} \leq \sum_{\forall \tau_j \in \gamma_i} \left( \left\lceil \frac{T^j}{T^i} \right\rceil + 1 \right) s_{i,j_{\text{max}}} \).

(11)

\[ \therefore: \zeta_i \text{ includes all jobs with higher priority than } \tau_i, \text{ and } \tilde{\tau}_i \text{ includes all jobs with lower priority than } \tau_i. \quad \therefore: (11) \text{ holds if } \forall \tau_i \]

\[
\frac{s_{i,j_{\text{max}}}}{s_{j_{\text{min}}}} \leq \frac{\left\lceil \frac{1}{m} \sum_{\forall \tau_j \in \gamma_i} \left( \left\lceil \frac{T^j}{T^i} \right\rceil + 1 \right) s^h_j \right\rceil} {\sum_{\forall \tau_j \neq \tau_i} \left( \left\lceil \frac{T^j}{T^i} \right\rceil + 1 \right)}
\]

(12)

From (10) and (12), Claim follows.

VII. EXPERIMENTAL EVALUATION

We now would like to understand how FBLT’s retry cost compares with competitors in practice (i.e., on average). Since this can only be understood experimentally, we implemented FBLT and the competitors and conduct experiments.

We used the ChronOS real-time Linux kernel [13] and the Rochester STM (RSTM) library [12] in our implementation. We implemented G-EDF and G-RMA schedulers in ChronOS, and modified RSTM to include implementations of FBLT, ECM, RCM, LCM, and PNF. For the retry-loop lock-free synchronization, we used a loop that reads an object and attempts to write to it using a CAS instruction. The task retries until the CAS succeeds. We used an 8 core, 2GHz AMD Opteron platform. The average time taken for one write operation by RSTM on any core is 0.0129653375μs, and the average time taken by one CAS-loop operation on any core is 0.0292546250 μs. We used four task sets consisting of 4, 5, 8, and 20 periodic tasks. Each task runs in its own thread and has a set of atomic sections. Atomic section properties are probabilistically controlled using three parameters: the maximum and minimum lengths of any atomic section within a task, and the total length of atomic sections within any task. Since lock-free synchronization cannot handle more than one object per atomic section, we first compare FBLT’s retry cost with that of lock-free (and other CMs) for one object per transaction. We then compare FBLT’s retry cost with that of other CMs for multiple objects per transaction.

Figure 1 shows the average retry cost for the 5 task set sharing one object. On the x-axis of the figures, we record 3 parameters x, y, and z. The x-axis is the ratio of the total length of all atomic sections of a task to the task WCET. The y-axis is the ratio of the maximum length of any atomic section of a task to the task WCET. The confidence level of all data points is 0.95. While Figure 1(a) includes all synchronization methods, Figure 1(b) excludes lock-free. From these figures, we observe that lock-free has the largest retry cost, as it provides no conflict resolution. FBLT has the largest retry cost among CMs, because transactions share only one object in this case. For multiple objects per transaction, PNF has an advantage over FBLT. However, PNF requires a-priori knowledge of all objects accessed by each transaction, whereas FBLT does not. Consequently, retry cost under PNF is a little shorter than that under FBLT. Experiments show that FBLT’s retry cost can be shorter than that under ECM, RCM, and LCM, and can be comparable to that of PNF’s as shown in Figure 2. PNF was designed to avoid transitive retry. Previous experiments compare retry cost of different CMs

![Fig. 1. Average retry cost (one object/transaction).](image-url)
in case of transitive retry. Figure 3 compares retry costs of different CMs in case of non-transitive retry. FBLT achieves shorter or comparable retry cost to other CMs including PNF. Similar trends were observed for the other task sets; those are omitted here due to space limitations.

VIII. CONCLUSIONS

Transitive retry increases transactional retry costs under ECM, RCM, and LCM. PNF avoids transitive retry by avoiding transactional preemptions. It avoids transitive retry cost by concurrently executing non-conflicting transactions, which are non-preemptive. However, PNF requires a-priori knowledge about objects accessed by each transaction. This is incompatible with dynamic STM implementations. Thus, we introduce the FBLT contention manager. Under FBLT, each transaction is allowed to abort for a no larger than a specified number of times. Afterwards, the transaction becomes non-preemptive. Non-preemptive transactions have higher priorities than other preemptive transactions and real-time jobs. Non-preemptive transactions resolve their conflicts using FIFO order. By proper adjustment of the maximum abort number of each transaction, we showed that FBLT’s schedulability is equal to or better than PNF.

Our experimental results show that FBLT has equal or shorter retry cost than ECM, RCM, and LCM. PNF requires a-priori knowledge of all objects accessed by each transaction. This is an advantage for PNF over FBLT. Consequently, retry cost under PNF is shorter than that under FBLT in case of transitive retry. Still, FBLT’s retry cost can be comparable to PNF’s. In case of no or low transitive retry, FBLT achieves shorter retry cost than other CMs including PNF. Future work includes choosing another criterion to resolve conflicts of non-preemptive transactions. Also, using feedback from the system to adjust maximum abort number of each transaction. Consequently, retry cost can be reduced over time.

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